The UCSD/SDSU Mathematics and Science Education
Doctoral Program Proudly Presents a Dissertation Defense:

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The Music of Mathematics: Toward a New Problem Typology

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Halmos once described problems and their solutions as “the heart of mathematics”. Following this line of thinking, one might naturally ask: “What, then, is the heart of problems?”. In this work, I attempt to answer this question using techniques from statistics, information visualization, and machine learning. I begin the journey by cataloging the features of problems delineated by the mathematics and mathematics education communities. These dimensions are explored in a large data set of students working thousands of problems at the Art of Problem Solving, an online company that provides adaptive mathematical training for students around the world. This analysis is able to concretely show how the fabric of mathematical problems changes across different subjects, difficulty levels, and students. Furthermore, it locates problems that stand out in the crowd – those that synergize cognitive engagement, learning, and difficulty.

This quantitatively-heavy side of the dissertation is partnered with a qualitatively-inspired portion that involves human scoring of 105 problems and their solutions. In this setting, I am able to capture elusive features of mathematical problems and derive a fuller picture of the space of mathematical problems. Using correlation matrices, principal components analysis, and clustering techniques, I explore the relationships among those features frequently discussed in mathematics problems (e.g., difficulty, creativity, novelty, affective engagement, authenticity). Along the way, I define a new set of uncorrelated features in problems and use these as the basis for a New Mathematical Problem Typology (NMPT). Grounded in the terminology of classical music, the NMPT works to quickly convey the essence and value of a problem, just as terms like “etude” and “mazurka” do for musicians.

Taken together, these quantitative and qualitative analyses seek to terraform the landscape of mathematical problems and, concomitantly, the current thinking about that world. Most importantly, this work highlights and names the panoply of problems that exist, expanding the myopic vision of contemporary mathematical problem solving.